

Statistics Exercises

Exercises 1

Characteristics of Distributions

- 1 The following frequency distribution represents the number of particles emitted by a radioactive source and detected by a Geiger counter in 40 consecutive periods of 1 minute.

No. of particles per minute	x :	0	1	2	3	4	5
No. of minutes (frequency)	f :	9	13	7	6	3	2

Calculate the mean and standard deviation for the distribution.

$$(n = 40, \bar{x} = 1.68, s = 1.44)$$

- 2 The red blood cell cholinesterase values (μ mole/min/ml) among 35 agricultural workers exposed to pesticides were as given in the following grouped frequency distribution.

RBC Cholinesterase (μ mole/min/ml)	6.0 - 7.9	8.0 - 9.9	10.0 - 11.9	12.0 - 13.9
freq	1	8	14	9
	14.0 - 15.9	16.0 - 17.9		
	2	1		

Calculate (i) the sample mean and (ii) the standard deviation.

$$(n = 35, \bar{x} = 11.29, s = 2.09)$$

- 3 For the frequency distribution of question 2 construct a cumulative frequency table, draw the ogive and from the ogive estimate

- (i) the median
- (ii) the semi-interquartile range

$$(\text{median} \cong 11.1, \text{s.i.q.r.} \cong 1.3)$$

- 4 Observations were taken on the percentage reduction in the growth of samples of a bacterial culture produced by two strains A and B of penicillin.

Strain A:	41.4	48.3	51.2	30.3	56.8	45.7
	51.7	62.4	36.8	57.3	47.6	
Strain B:	10.7	16.3	27.2	18.7	21.9	
	13.2	15.1	8.3	25.6	23.1	

For both strain A and strain B observations calculate (i) \bar{x} (ii) s (iii) the coefficients of variation.

(Strain A: $\bar{x} = 48.14$, $s = 9.40$, $v = 19.5\%$
Strain B: $\bar{x} = 18.01$, $s = 6.38$, $v = 35.4\%$)

Exercises 2

Probability

- 1 Assume 25% of the population has blood group B and 40% have blood group O. If someone is selected at random from this population what is the probability that he or she is either blood group B or blood group O?
(0.65)
- 2 Assume 40% of the population has blood group O and 50% have blue eyes. If one person is selected at random from the population find the probability that the person chosen is blood group O and has blue eyes.
(0.2)
- 3 A certain disease has a mortality rate of 75%. Two patients suffering from the disease are selected at random. What is the probability that at least one of them will recover?
(0.4375)
- 4 Among 10 patients, 7 are blood group O and 3 are blood group B. Two patients are selected at random to undergo a particular test. What is the probability that
 - a) both are group O
 - b) both are group B
 - c) they are of the same blood group
 - d) they are of different blood groups
(0.4667, 0.0667, 0.5333, 0.4667)
- 5 A piece of hospital equipment will function only if 3 components A, B and C are all working. The probability of A failing during one year is 0.05, that of B failing is 0.15 and that of C is 0.10. What is the probability that the equipment fails before the end of the year?
(0.27325)
- 6 After a particular course of treatment the recovery rate from a certain ailment is 3 out of every 4 men and 4 out of every 7 women. A man and his wife both suffering from this ailment undergo the course of treatment. What is the probability that
 - a) both recover
 - b) neither recovers
 - c) at least one recovers
(0.4286, 0.1071, 0.8929)

- 7 Two men A and B toss a coin alternately on the understanding that the first one to obtain "heads" wins the game. A tosses first. Show that his probability of winning is $\frac{2}{3}$.

(Hint: use the fact that the sum of an infinite geometric progression :

$$a + ar + ar^2 + ar^3 + ar^4 \dots = \frac{a}{1-r} \quad (0 < r < 1).$$

Exercises 3

Binomial Distribution

- 1 The probability of a treatment for a particular disease alleviating all signs and symptoms is 0.6. Four patients are treated, determine the probabilities that
 - a) there are no cures (0.0256)
 - b) there are exactly 2 cures (0.3465)
 - c) there are at least 3 cures (0.4752)
- 2 Out of 800 families with 5 children how many would you expect to have
 - a) 3 boys (250)
 - b) 5 girls (25)
 - c) either 2 or 3 boys (500)
- 3 What is the probability of getting 9 exactly once in 3 throws of a pair of fair dice.
(0.2634)
- 4 Suppose that it is known in a certain population 10% of the population is colour blind. If a random sample of 50 people is drawn from this population use tables to find the probability that
 - a) 5 or more will be colour blind (0.5688)
 - b) 4 or fewer will be colour blind (0.4312)
 - c) between 6 and 10 will be colour blind (0.3745)
- 5 Suppose that the probability of recovery from a certain disease is 0.4. If 10 people are stricken with the disease (assume this to be a random sample), what is the probability that
 - a) 3 or more will recover (0.8327)
 - b) exactly 5 will recover (0.2007)
- 6 An area of a laboratory housing temperature sensitive equipment has temperature sensors installed which give an alarm when the temperature rises above a certain level. Individual sensors have only a 95% chance of giving a warning when the danger level is reached. Consequently, several sensors are fitted and they all function independently.

If 3 devices are fitted what is the probability that
 - a) exactly 2 function correctly
 - b) at least 2 function correctly?

How many should be fitted to ensure at least a 99.99% chance of getting a warning signal when necessary?

(a) 0.1353 b) 0.9927 :4)

Exercises 4

Poisson Distribution

- 1 In a study of a certain aquatic organism, a large number of samples were taken from a pond, and the number of organisms in each sample counted. The average number of organisms per sample was found to be 2. Assuming the number of organisms to be distributed according to a Poisson distribution find the probability that the next sample taken will contain:

- a) 1 or more organisms (0.8647)
- b) exactly 3 organisms (0.1804)
- c) fewer than 5 organisms (0.9473)

- 2 A set of 100, 0.1 minute radiological counts were taken from a single source giving the following results.

Count x:	0	1	2	3	4	5+
Observed frequency:	11	20	28	24	12	5

Find the expected frequencies of a Poisson distribution having the same mean as this distribution.

(11.0, 24.2, 26.8, 19.7, 10.9, 7.4)

- 3 Under microscopic investigation, on the average 5 particular micro-organisms are found on a one square centimetre specimen. If it is assumed that the micro-organisms are randomly distributed, what are the probabilities of finding a specimen with
- a) less than 3 organisms (0.1246)
 - b) exactly 5 (0.1755)
 - c) more than 6 (0.2378)
 - d) 2 or 3 (0.2246)
- 4 An ecologist wanted to determine whether goldenrod plants containing one or more galls tend to be randomly distributed or clumped. He hypothesised that clumping might suggest that the wasps that produce the galls range only short distances from the point of emergence. He selected a field containing the plants and randomly threw 100 quadrats throughout the field. Counting the number of gall-bearing plants in each quadrat resulted in the following frequency distribution.

Observed frequency f	No of plants with galls per quadrat x
39	0
24	1
19	2
14	3
4	4+

Comment on the hypothesis made by the Ecologist by fitting a Poisson distribution to the data.

(30.1, 36.1, 21.7, 8.7, 3.4)

- 5 In a certain population the probability of an individual having colour blindness is 0.02. If 100 persons are selected from this population what is the probability that
- a) 2 individuals will be colour blind (0.2707)
 - b) at least 2 individuals will be colour blind (0.5940)
 - c) none will be colour blind (0.1353)

Exercises 5

Normal Distribution

- 1 Find the areas under the Standard Normal curve in the following cases:
 - a) between $z = 0$ and $z = 1.2$
 - b) between $z = -0.68$ and $z = 0$
 - c) between $z = -0.35$ and $z = 1.9$
 - d) between $z = 0.46$ and $z = 2.2$
 - e) to the left of $z = 0.6$
 - f) to the right of $z = -1.13$

- 2 If $x \sim N(0.36, 0.04^2)$ find
 - a) $P(x < 0.42)$ (0.9332)
 - b) $P(x < 0.3)$ (0.0668)
 - c) $P(x > 0.32)$ (0.8413)
 - d) $P(0.36 < x < 0.44)$ (0.4772)
 - e) $P(0 < x < 0.5)$ (0.9998)
 - f) x_0 if $P(x < x_0) = 0.65$ (0.3754)
 - g) x_0 if $P(x < x_0) = 0.35$ (0.3446)

- 3 Assume that the weights of a group of individuals are approximately Normally distributed with mean 64 kg and standard deviation 12 kg, what is the probability that a person chosen at random from the group will weigh between 43 and 76 kg?
(0.8012)

- 4 If the capacities (cm^3) of the cranial cavity of a certain population are approximately $N(1400, 125^2)$ find the probability that someone chosen at random from this population will have a cranial cavity capacity:
 - a) greater than $1450 cm^3$ (0.3446)
 - b) less than $1350 cm^3$ (0.3446)
 - c) between $1300 cm^3$ and $1500 cm^3$ (0.5762)

- 5 Assuming that the total cholesterol values (in mg/100 ml) are approximately $N(200, 20^2)$, find the probability that an individual chosen at random will have a cholesterol value
 - a) between 180 and 200 mg/100 ml (0.3413)
 - b) greater than 225 mg/100 ml (0.1056)
 - c) less than 150 mg/100 ml (0.00621)
 - d) between 205 and 215 mg/100 ml (0.1747)

- 6 Refer to question 3. If 5 individuals are selected from this population, find the probability that:
- a) none will weigh between 43 and 76 kg (0.00031)
 - b) at least one will weigh between 43 and 76 kg (0.99969)
 - c) exactly 2 will weigh between 43 and 76 kg (0.05043)
- 7 A random variable has a Normal distribution with standard deviation 1.4. Determine the mean of the distribution if the probability that the random variable will take a value less than 59 is 0.9772.
- (56.2)